Game Forms versus Social Choice Rules as Models of Rights

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ABSTRACT

The paper begins by defining multi-valued game forms which generalize social choice rules. It also defines the effectiveness relations that represent the rights induced by such game forms. When one-way instead of two-way rights are allowed, it also demonstrates yet further difficulties in finding a social choice rule to respect all rights. Finally, to deal with the problem that game forms typically contain arbitrary features of no consequence to a society or to its individual members, it is also suggested that one should define both social choice and individual values over sets consisting of "consequentially equivalent" classes of strategic game forms.

Modelling Rights

1. Introduction

When Amartya Sen (1970a) first introduced the idea of rights into formal social choice theory, he did so through the fairly standard apparatus of a *social choice rule* (or SCR) — see also Sen (1970b). By contrast, especially since the provocative work of Peter Gärdenfors (1981), Robert Sugden (1981, 1985, 1986), and others on this issue, more recent writers have often preferred to consider *game forms*. Some of the relationships, as well as the advantages and disadvantages of each approach, have also been discussed recently by Riley (1989, 1990), Gaertner, Pattanaik, and Suzumura (1992), Pattanaik and Suzumura (1992), Sen (1992), and Hammond (1994a).

Rather than go over all these arguments once again, I will suggest a generalized model of rights that embeds both the SCR and game form approaches as special cases. Indeed, there is a sense in which the *multi-valued game form* model I propose shows that "direct" game forms, in which individuals' strategies are direct reports of their preferences, are really implicit in Sen's original SCR model of rights. After some notation and definitions have been set out in the preliminary Section 2, this is the topic of Section 3 of the paper.

Next, Sections 4 and 5 discuss some of the different ways in which a multi-valued game form can be said to respect rights, or to give individuals the power to exercise their rights. Indeed, this is probably the major difference between the SCR and game form approaches. Specifically, for arbitrary multi-valued game forms, Section 5 suggests defining rights in terms of "effectiveness relations." These generalize the effectiveness functions which Rosenthal (1972) first introduced into cooperative game theory. Note that Gärdenfors (1981) also modelled rights in terms of what could be regarded as effectiveness functions.

In Hammond (1982, 1994a), it was shown how the restriction to "privately oriented" preferences allows "two-way" rights to be respected. Moreover, the conflict between respect for rights and Pareto efficiency disappears entirely. Indeed, as Coughlin (1986) has pointed out, Pareto efficiency even requires respect for rights in this special case. But this is only because, by assumption, no externalities arise in the exercise of individual or group rights. Section 6 notes how this particular restriction leads to severe difficulties when individuals and groups are both allowed to have one-way rights.

Accordingly, except in special cases, society has no way of avoiding the choice of which rights to respect and which to violate. Now, one can think of choosing a configuration of rights as being equivalent to choosing a game form that induces those rights. In the past, an objection to this approach has been that the typical (strategic) game form includes several arbitrary features, such as the names of the strategies, etc. To avoid this arbitrariness, Section 7 considers equivalence classes of "consequentialist" reduced strategic game forms, along the lines of invariance requirements in classical normal form game theory. In the end it appears reasonable to consider both social choice and individual values as being defined over sets of such equivalence classes.

Finally, Section 8 contains a brief concluding assessment.

2. Preliminary Notation and Definitions

As usual in social choice theory, I shall suppose that there is a fixed domain or *underlying set* X of conceivable social states, and a fixed finite set of individuals N with variable *preference orderings* R_i ($i \in N$), which are members of the set $\mathcal{R}(X)$ of all logically possible complete and transitive binary relations defined on X. Let P_i and I_i ($i \in N$) denote the corresponding *strict preference* and *indifference* relations, respectively; these must also be transitive. A *preference profile* $R^N = \langle R_i \rangle_{i \in N}$ is a listing of preference orderings, one for each individual in society. Such a profile belongs to the Cartesian product space $\mathcal{R}^N(X) := \prod_{i \in N} \mathcal{R}_i(X)$, where each $\mathcal{R}_i(X)$ is a copy of $\mathcal{R}(X)$.

In social choice theory, it is usual to let the feasible set A range over the whole space $\mathcal{F}(X)$ of all possible non-empty finite feasible subsets of X. Here, however, I shall follow the mechanism design literature, and specifically Dasgupta, Hammond, and Maskin (1979), in treating the feasible set $A \in \mathcal{F}(X)$ as fixed. A *social choice rule* (SCR) therefore specifies, for the given feasible set A and for each preference profile $R^N \in \mathcal{R}^N(X)$, a non-empty subset $C_A(R^N) \subset A$.

For each preference profile R^N and each non-empty $G \subset N$, let $P_G(R^N)$ and $P^*_G(R^N)$ denote the corresponding *strict* and *strongly strict group preference relations* defined for all pairs $a, b \in X$ by

$$a \; P_G(R^N) \; b \iff \forall i \in G : a \; P_i \; b$$

$$\text{and} \quad a \; P^*_G(R^N) \; b \iff \{[\forall i \in G : a \; R_i \; b] \;\&\; [\exists h \in G : a \; P_h \; b]\}.$$

Because each individual's preference relation is transitive, so are the relations $P_G(R^N)$ and $P^*_G(R^N)$. In particular, the *weak* and the *strict Pareto dominance relations* $P_N(R^N)$ and $P^*_N(R^N)$ are both transitive.

Finally, two further pieces of notation. First, given any Cartesian product set of the form $V^N = \prod_{i \in N} V_i$ and any subset $G \subset N$, write V^G for the restricted Cartesian product set $\prod_{i \in G} V_i$, with typical member $v^G = \langle v_i \rangle_{i \in G}$. Second, write V_{-i} for the product set $V^{N \setminus \{i\}} = \prod_{h \in N \setminus \{i\}} V_h$ with V_i omitted, and v_{-i} for the typical member $v^{N \setminus \{i\}} = \langle v_h \rangle_{h \in N \setminus \{i\}}$ of V_{-i}.

3. Social Choice Rules and Multi-valued Game Forms

Sen's (1970a, b) original representation of minimal liberty took a highly specific form. In a society with a finite set of individuals N, it requires at least two individuals $j, k \in N$ to be given the right to be decisive over at least one personal issue each in the form of a single pair of alternative social states — say, a_j versus b_j, and a_k versus b_k. Here, the subset $\{ a_j, b_j, a_k, b_k \} \subset A$ has to have at least three distinct members. Thus, there has to be an SCR which respects at least the minimal rights of j and k in the sense that:

$$a_j \; P_j \; b_j \Longrightarrow b_j \notin C_A(R^N); \quad a_k \; P_k \; b_k \Longrightarrow b_k \notin C_A(R^N).$$

Sen showed how such minimal liberty was incompatible with even weak Pareto efficiency, in general. Then Batra and Pattanaik (1972) demonstrated a similar incompatibility for "minimal federalism," when different groups were given the right to determine issues by the unanimous expression of a strict preference. Not long afterwards, Gibbard (1974) showed how it might be impossible to respect individual rights at all, even if one did not insist on Pareto efficiency.

Though at the time they seem not to have been considered as such, the SCRs used in this early work could be regarded as multi-valued extensions of *direct mechanisms*, in the terminology we used in Dasgupta, Hammond and Maskin (1979) that seems to have become widely accepted since. That is, the individuals $i \in N$ could be regarded as players of a multi-valued game form whose strategies are direct reports of their own preferences. The SCR considered by Sen then describes what would happen, for each fixed feasible set $A \in \mathcal{F}(X)$, if preferences were reported sincerely.

Gibbard (1974) also considered what would happen in a game where individuals could make strategic decisions to waive their rights in some instances. Actually, Gibbard's work served to suggest more than just how strategic aspects could play a role when individuals were accorded certain rights. It also suggested the need to consider *indirect mechanisms*, meaning extended game forms in which the strategy sets could be more general than reports of preferences. This, along with Fine's (1975) early exploration of the relationship between Sen's paradox and prisoners' dilemma, may have done much to prompt the much more explicit game-theoretic approach to be found in later works such as those by Gärdenfors (1981), Sugden (1981, 1985, 1986), and Seidl (1986).

	single-valued	multi-valued
direct	singleton social choice rules	social choice rules
indirect	game forms	multi-valued game forms

Table 1. The four kinds of social choice rule or game form.

As a way of generalizing both the SCR and the game form models of rights, I shall follow a suggestion due to Peleg (1984b) for cooperative games. For every fixed feasible set $A \in \mathcal{F}(X)$, consider a *multi-valued game form* $\langle N, S^N, \Gamma_A(s^N) \rangle$ in which: (i) N is the set of individual players; (ii) $S^N := \prod_{i \in N} S_i$ is the set of possible strategy profiles; (iii) $\Gamma_A(s^N) \subset A$ is the non-empty social choice set, defined for every strategy profile $s^N = \langle s_i \rangle_{i \in N} \in S^N$. Note that the SCR approach to rights postulates a direct game form, in which each individual's strategy set S_i consists of the set $\mathcal{R}(X)$ of all logically possible preference orderings over the domain X. The same approach also ignores the incentives which individuals generally have to misrepresent their true preferences. On the other hand, the usual game form approach to rights postulates a function with single values $g_A(s^N) \in A$ instead of the (generally multi-valued) correspondence $\Gamma_A(s^N)$.

After including the most special case of singleton (direct) social choice rules, the four different kinds of social choice rule or game form are described in Table 1. Note in particular that a multi-valued game form is really equivalent to an indirect social choice rule.

4. Rights-Respecting Social Choice Rules

Given any pair $a, b \in X$, the (non-empty) group $G \subset N$ is said to be *decisive for a over b* if, whenever $a, b \in A$ and the profile R^N is such that $a\ P_G(R^N)\ b$, then $b \notin C_A(R^N)$. In other words, group G should be able to veto b if a is available and the members of G unanimously prefer a to b. The same definition applies to individuals $i \in N$, of course, taking $G = \{i\}$.

It is usual to regard the (one-way) rights of each group $G \subset N$ as being represented by a (possibly empty) collection $D_G \subset X \times X$ of ordered pairs for which G is supposed to be decisive. Of course, this set D_G can be regarded as the graph of a binary preference relation; this being so, D_G can be called a *one-way rights relation* without undue confusion. It will be assumed that D_G is irreflexive — i.e., that there is no $x \in X$ with $x\ D_G\ x$. In case D_G is symmetric, it will be called a *two-way rights relation.* On the whole, the literature has typically considered two-way rights. The example of Section 7 should serve to show, however, that this can be unduly restrictive. Accordingly, most results in this paper require only one-way rights, though the corollary in Section 6 will require two-way rights relations.

Let $\mathcal{G}$ denote the collection of groups G having non-trivial rights relations D_G. In case only individuals have rights, it will be true that $\mathcal{G} \subset \{\, \{i\} \mid i \in N \,\}$. But here I allow the possibility that groups may have rights, and also that some or all individuals may have no rights. Usually D_i instead of $D_{\{i\}}$ will be used to indicate individual i's rights relation.

In what follows, it will be assumed that a particular *rights profile* $D^{\mathcal{G}}$ of irreflexive rights relations $\langle D_G \rangle_{G \in \mathcal{G}}$ has been specified, for some set $\mathcal{G} \subset 2^N$ of groups (and individuals) with rights. Note that, if G' is a proper subset of G, then G will be decisive over $\{\, a, b \,\}$ whenever G' is.

Finally, say that the SCR $C_A(R^N)$ *respects the rights profile* $D^{\mathcal{G}}$ if, whenever $a, b \in A$ with $a\ D_G\ b$, then G is decisive for a over b.

5. Rights Induced by Multi-valued Game Forms

Let $\langle N, S^N, \Gamma_A \rangle$ be a multi-valued game form, with $\Gamma_A : S^N \twoheadrightarrow A$. Also, let $G \subset N$ be a particular group of individuals, and $Y, Z \subset X$ a disjoint pair of non-empty sets of social states. In this case, say that G is *α-effective* for Y over Z, and write $Y\ D_G^\alpha\ Z$, iff there exists $\bar{s}^G \in S^G$ such that, for all $s^N \in S^N$ satisfying $\Gamma_A(s^N) \cap Z \neq \emptyset$, one has $\Gamma_A(\bar{s}^G, s^{N\setminus G}) \subset Y$. In other words, $Y\ D_G^\alpha\ Z$ requires that group G alone always has the power to veto any outcome in the set Z and restrict the possible social choice outcome to the set Y instead, no matter what fixed strategies $s^{N\setminus G}$ the other individuals choose. One says then that D_G^α is group G's *α-rights relation induced* by the multi-valued game form.

Suppose that, because of a mistake by the members of group G or for some other reason, some social state $b \in Z$ is still a social outcome that results from the game form, even though $Y\ D_G^\alpha\ Z$ and $a\ P_i\ b$ for all $i \in G$ and for all $a \in Y$. Yet it is illegitimate for G to claim any rights violation, because the game form did give G the opportunity to veto the outcome b.

On the other hand, say that G is *β-effective* for Y over Z, and write $Y\ D_G^\beta\ Z$, iff for all $s^N \in S^N$ satisfying $\Gamma_A(s^N) \cap Z \neq \emptyset$, there exists $\bar{s}^G(s^{N\setminus G}) \in S^G$ such that $\Gamma_A(\bar{s}^G(s^{N\setminus G}), s^{N\setminus G}) \subset Y$. Unlike with α-effectiveness, β-effectiveness allows the group G to choose its strategy $\bar{s}^G(s^{N\setminus G})$ as if responding to the expectation that the other players will choose the appropriate components of $s^{N\setminus G}$. One says then that D_G^β is group G's *β-rights relation induced* by the multi-valued game form. Evidently $Y\ D_G^\beta\ Z$ implies $Y\ D_G^\alpha\ Z$, but the converse is generally false. Of course, this distinction between α- and β-effectiveness is precisely analogous to the distinction between the α- and β-characteristic functions of a cooperative game without side payments, as well as to the corresponding distinction between the α- and β-cores of such a game. See, for example, Aumann and Peleg (1960) and also Aumann (1967).

In interpreting $Y\ D_G^\alpha\ Z$ or $Y\ D_G^\beta\ Z$ as indicating that the game form gives G the right to Y over Z, it is important that the members of Y and Z consist of completely described social states or consequences. In particular, everything of relevance to the individuals in group G must be included in each consequence description. For example, if G's members can only avoid the damaging effects of other individuals' polluting activities by paying them to desist, then G does *not* have the right to avoid pollution; in fact, G only has the right to achieve less pollution at the cost of less money. Similarly, the right to be free of tobacco smoke without having to ask is different from the right to ask others not to smoke; at least, this is true if non-smokers are troubled by the need to ask, because then the need to ask becomes part of the relevant consequence.[1]

Also note that the above definitions are of binary effectiveness relations, rather than the "effectiveness" or "effectivity functions" considered by Rosenthal (1972), Moulin and Peleg (1982), Moulin (1983), Peleg (1984a, b), and Deb (1990, 1994).[2] One reason for this is the closer analogy with the binary rights relations for social choice rules considered by Sen and many later writers. A second reason is greater generality. For, given the feasible set $A \in \mathcal{F}(X)$ and the subset $Y \subset A$, observe that G is *α-effective for the set* Y iff $Y \; D_G^\alpha \; A \setminus Y$, and similarly for β-effectivity. Thus, it is easy to infer effectiveness functions from effectiveness relations, but the converse is not true at all.

Furthermore, the binary effectiveness relations D_G^α and D_G^β are defined without reference to an equilibrium of any kind. This is crucial. Indeed, were we to consider the rights to consequences that emerge from an equilibrium, we would be back with direct SCRs and direct rights to consequences. For if $E : \mathcal{R}^N \twoheadrightarrow S^N$ is an equilibrium correspondence, then there is an *equivalent direct* SCR $C_A : \mathcal{R}^N \twoheadrightarrow A$ defined by $C_A(R^N) := \Gamma_A(E(R^N))$ for all $R^N \in \mathcal{R}^N$ — cf. Dasgupta *et al.* (1979).

For the special case when $\Gamma_A : S^N \twoheadrightarrow A$ is a single-valued function $g_A : S^N \to A$ and also $Y = \{a\}$ and $Z = \{b\}$, the above definitions become much simpler. In fact, then $\{a\} \; D_G^\alpha \; \{b\}$ iff there exists $\bar{s}^G \in S^G$ such that, for all $s^N \in S^N$ satisfying $g_A(s^N) = b$, one has $g_A(\bar{s}^G, s^{N \setminus G}) = a$. On the other hand, $\{a\} \; D_G^\beta \; \{b\}$ iff, for all $s^N \in S^N$ satisfying $g_A(s^N) = b$, there exists $\bar{s}^G(s^{N \setminus G}) \in S^G$ such that $g_A(\bar{s}^G(s^{N \setminus G}), s^{N \setminus G}) = a$. Indeed, the "induced rights relation" considered in Hammond (1994) is exactly the relation D_G^β, applied to single-valued game forms and to individual elements of the feasible set A.

[1] I am grateful to Jerry Kelly for a discussion which showed the need to emphasize this point.

[2] Deb also defines "waiver" functions, and gives a much more extensive discussion of how both effectivity and waiver functions can be used to represent important aspects of rights.

6. Rights Oriented Preferences

What rights relations can be respected by a direct social choice rule? A famous example in the article by Gibbard (1974) was the first to show that not all can be. This issue was also considered by Farrell (1976). Then Suzumura (1978) defined the profile D^N of individual rights relations to be "coherent" provided that, for an unrestricted domain of preference profiles R^N, there exists an SCR $C_A(R^N)$ which respects those rights for every feasible set $A \in \mathcal{F}(X)$. However, Gibbard's example serves to show how very restrictive is the assumption of coherence.

Thereafter, Blau (1975), Seidl (1975), Farrell (1976), and Breyer (1977) are just a few of the early works suggesting that restricting the domain of preferences could allow rights to be respected. In Hammond (1982, 1994a), it was shown that restricting attention to "privately oriented" preferences allows two-way rights to be respected, and that Pareto efficiency even requires such rights to be respected. In the case of one-way rights, the obvious corresponding restriction is the following. Say that individual preferences are *rights oriented* provided that, for every $i \in N$, whenever $a \; D_G \; b$ for some G satisfying $i \notin G \in \mathcal{G}$, it is true that $a \; R_i \; b$. Thus, i's rights oriented preferences defer to the rights held by any group G to which i does not belong by expressing an appropriate weak preference for G to be allowed to exercise its right. By assumption, therefore, no externalities can possibly arise in the exercise of individual or group rights — cf. Hillinger and Lapham (1971).

Of course, in case both $a \; D_G \; b$ and $b \; D_G \; a$ for some G satisfying $i \notin G \in \mathcal{G}$, rights oriented preferences must satisfy $a \; I_i \; b$. This is exactly what is required of privately oriented preferences when all rights are two-way.

The following is the obvious counterpart of Theorem 1 in Hammond (1994a), which generalizes a result due to Coughlin (1986):

THEOREM 1. *Suppose that, for the given rights profile $D^{\mathcal{G}}$, the domain of allowable preference profiles R^N is restricted to rights oriented preferences. Then the social choice rule $C_A(R^N)$ satisfies the strict Pareto rule for all $A \in \mathcal{F}(X)$ only if it respects both individual and group rights.*

PROOF: Suppose that the feasible set $A \in \mathcal{F}(X)$, social states $a, b \in A$, and the group $G \in \mathcal{G}$ are such that $a \; D_G \; b$. Suppose too that the rights oriented preference profile R^N satisfies $a \; P_G(R^N) \; b$. Since R^N is rights oriented, it follows that $a \; R_i \; b$ for all $i \in N \setminus G$, and so $a\ P^*_N(R^N)\ b$. If the SCR is strictly Paretian, therefore, it must be true that $b \notin C_A(R^N)$, proving that G is decisive for a over b. So all rights are respected by any strictly Paretian SCR. ▮

Indeed, when preferences are rights oriented, then liberalism "does not demand anything that the Pareto principle does not also demand," as Sen (1971, pp. 1406–7) puts it in his response to Hillinger and Lapham (1971).

However, in the case of one-way rights, it seems extremely unreasonable to require preferences to be rights oriented. To show this, I shall use once again a variation of an example considered by Gibbard (1974, p. 398) and Gärdenfors (1981) which was apparently suggested by Gilbert and Sullivan's operetta *Trial by Jury*. Society N consists of three individuals — A (Angelina), E (Edwin), and J (the male judge). The domain X consists of the three social states $0, e, j$, where 0 indicates that Angelina marries neither of the two men, e that she marries Edwin, and j that she marries the judge. It is natural to give each of the two couples $\{A, E\}$ and $\{A, J\}$ the one-way group right to marry, and to give each individual the one-way right to avoid marrying someone they would rather not. Thus, the one-way rights relations can be expressed as:

$$e\ D_{AE}\ 0; \quad j\ D_{AJ}\ 0; \quad 0\ D_A\ e; \quad 0\ D_E\ e; \quad 0\ D_A\ j; \quad 0\ D_J\ j.$$

In this case, it is easily seen that restricting preferences to be rights oriented requires:

$$0\ R_A\ e; \quad 0\ R_A\ j; \quad 0\ I_E\ j; \quad 0\ I_J\ e.$$

Hence, in the event that Angelina does not marry him, each man is required to be indifferent about whether she marries the other. This certainly has some ethical appeal. However, it is deeply troubling that Angelina cannot have any strict preference for marrying either man as opposed to neither of them; this is what follows from not allowing her to express a strict preference that conflicts with the possible desire of either man not to marry Angelina. It seems that other ways of satisfying rights while escaping Pareto inefficiency will have to be found when individuals' and groups' one-way rights come into conflict in this way.

At this point, it is worth emphasizing once again how these and other related negative results bring out the difficulty there is in satisfying all individual and group rights, even if appropriate restrictions are placed on individuals' preferences. It seems inevitable, therefore, that any SCR is bound to respect only some rights, leaving others to be violated. As Hammond (1994a) and Pattanaik and Suzumura (1992) both discuss, it then becomes important to have an explicit framework for discussing the social choice of which rights to respect.[3] The SCR approach does allow this, but limits attention to direct multi-valued game forms, even though the rights induced by an appropriate indirect multi-valued game form may often be both more extensive and ethically superior to any that an SCR can respect.

7. Consequentially Equivalent Strategic Game Forms

As pointed out in Section 1, a typical (strategic) game form includes several arbitrary features, such as the names of the strategies, etc. Yet such arbitrariness is easily circumvented by considering suitable equivalence classes of game forms with the same "consequentialist" reduced normal form. The concept of "reduced normal form" set out below will be the weaker version due to Dalkey (1953) rather than the stronger version used by Thompson (1952) and by Kohlberg and Mertens (1986). An important reason is that the latter themselves provide (p. 1019, game Δ_5) an example illustrating that consequentialist reduced normal form invariance, in their stronger sense, is not always possible when subgame perfect equilibria are being considered.

So, let $\Gamma = \langle N, S^N, \Gamma_A(s^N) \rangle$ and $\tilde{\Gamma} = \langle N, \tilde{S}^N, \tilde{\Gamma}_A(s^N) \rangle$ be two multi-valued game forms, each with the same set of players N and the same feasible set A of social states. These two are obviously identical when $S_i = \tilde{S}_i$ for all $i \in N$ and $\Gamma_A(s^N) = \tilde{\Gamma}_A(s^N)$ for all $s^N \in S^N = \tilde{S}^N$. But even when these conditions are not true, the two game forms may effectively be equivalent because one or more players' strategies have simply been replicated, or because Γ and $\tilde{\Gamma}$ differ only in the way in which strategies are labelled. This motivates the following definitions.

First, the two game forms $\Gamma = \langle N, S^N, \Gamma_A(s^N) \rangle$ and $\tilde{\Gamma} = \langle N, \tilde{S}^N, \tilde{\Gamma}_A(s^N) \rangle$ are said to be *(strategically) equivalent* if there is a one-to-one product correspondence $\rho^N : S^N \leftrightarrow \tilde{S}^N$ that satisfies $\rho^N(s^N) \equiv \prod_{i \in N} \rho_i(s_i)$ for some collection $\rho_i : S_i \leftrightarrow \tilde{S}_i$ $(i \in N)$ of individual one-to-one correspondences, and also has the property that $\tilde{\Gamma}_A(\rho^N(s^N)) = \Gamma_A(s^N)$ for all

[3] Early on, Hillinger and Lapham (1971) were among those to suggest, at least implicitly, that such a framework might be necessary. They did not put forward anything like a formal framework, however.

$s^N \in S^N$. Because ρ^N is a one-to-one correspondence, this implies that $\Gamma_A((\rho^N)^{-1}(\tilde{s}^N)) = \tilde{\Gamma}_A(\tilde{s}^N)$ for all $\tilde{s}^N \in \tilde{S}^N$.

A second form of equivalence treats all repetitions of "hyper-rows" of the payoff matrix as irrelevant. So there is an equivalence class of game forms which all share the same reduced normal form, without any such repetitions. Indeed, suppose the two game forms $\Gamma = \langle N, S^N, \Gamma_A(s^N)\rangle$ and $\tilde{\Gamma} = \langle N, \tilde{S}^N, \tilde{\Gamma}_A(s^N)\rangle$ are such that $S_i \subset \tilde{S}_i$ for all $i \in N$, while $\Gamma_A(s^N) = \tilde{\Gamma}_A(s^N)$ whenever $s^N \in S^N \subset \tilde{S}^N$. Moreover, suppose that for every player $i \in N$ and every strategy $\tilde{s}_i \in \tilde{S}_i \setminus S_i$, there exists some alternative $s^i \in S_i$ such that $\Gamma_A(s^i, s_{-i}) = \tilde{\Gamma}_A(\tilde{s}^i, s_{-i})$ for all $s_{-i} \in S_{-i}$. Then $\tilde{s}^i$ simply duplicates s^i in all its possible consequences, so $\tilde{s}^i$ may as well be omitted from the strategy set S^i. Since this is true for all $\tilde{s}_i \in \tilde{S}_i \setminus S_i$ by hypothesis, it follows that $\tilde{\Gamma}$ is effectively equivalent to Γ.[4]

With this in mind, consider the obvious way of reducing as far as possible the strategy sets of the game form $\Gamma = \langle N, S^N, \Gamma_A(s^N)\rangle$. Begin by saying that player i's two strategies $s_i, \bar{s}_i \in S_i$ are *(consequentially) equivalent* if $\Gamma_A(s_i, s_{-i}) = \tilde{\Gamma}_A(\bar{s}_i, s_{-i})$ for all $s_{-i} \in S_{-i}$. This is obviously an equivalence relation, so let Σ_i be the corresponding collection of equivalence classes, and write $\sigma_i(s_i)$ for the unique equivalence class containing the particular strategy s_i. Let $\Sigma^N := \prod_{i \in N} \Sigma_i$, with typical member σ^N. Note that, whenever $\sigma^N(s^N) = \sigma^N(\bar{s}^N)$ because s_i and $\bar{s}_i$ are in the same equivalence class for each $i \in N$, then $C(A, s^N) = C(A, \bar{s}^N)$. Hence, there is a well defined outcome function $\hat{\Gamma}_A(\sigma^N)$ satisfying $\hat{\Gamma}_A(\sigma^N(s^N)) := \Gamma_A(s^N)$ for all $s^N \in S^N$. The result of this construction is the *reduced normal form* $\hat{\Gamma} = \langle N, \Sigma^N, \hat{\Gamma}_A(\sigma^N)\rangle$ with the property that, for all $i \in N$ and every disjoint pair $\sigma_i, \bar{\sigma}_i \in \Sigma_i$, there exists $\sigma_{-i} \in \Sigma_{-i}$ for which $\hat{\Gamma}_A(\sigma_i, \sigma_{-i}) \neq \hat{\Gamma}_A(\bar{\sigma}_i, \sigma_{-i})$.

Finally, the two game forms $\Gamma = \langle N, S^N, \Gamma_A(s^N)\rangle$ and $\tilde{\Gamma} = \langle N, \tilde{S}^N, \tilde{\Gamma}_A(s^N)\rangle$ are said to be *(reduced normal form) equivalent* if their corresponding reduced normal forms are strategically equivalent, for some collection $\rho_i : \Sigma_i \leftrightarrow \hat{\Sigma}_i$ $(i \in N)$ of one-to-one correspondences between their respective reduced strategy sets. It is this notion of equivalence that eliminates irrelevant features such as the re-labelling or duplication of strategies.

[4] Deb (1990, 1994), in his discussion of closely related issues, assumes that all such repeated hyper-rows are simply deleted from the game form.

8. Concluding Assessment

Social systems *are* game forms, possibly even single-valued, from which social states or consequences emerge as a result of individuals' strategic decisions. This paper has pointed out that multi-valued game forms are rather natural extensions of social choice rules. It has recalled the difficulties in having a social choice rule respect rights. This suggests the impossibility of avoiding a choice of what rights to satisfy.

Of course, there are difficulties in modelling rights *as* game forms. These difficulties seem greater for multi-valued game forms, which is one good reason why previous writers have preferred to work with single-valued game forms. Still, one can discuss induced rights *within* game forms, and even ask which game form models of a social system induce the most satisfactory configurations of individual and group rights. This is the approach which seems to me most worth pursuing in future work on this subject.

Finally, let me return to the main subject and summarize the contrasts between the two approaches to rights. In game forms, induced rights concern strategies directly, and social states or consequences only indirectly. Indeed, rights to consequences in game forms are often contingent, depending on other individuals making suitable strategic decisions. Whereas in social choice rules, rights are to consequences directly, and these rights are non-contingent — provided that the social choice rule does manage to respect them, which can be problematic.

The main difference, therefore, is the indirect and contingent respect for rights that game forms typically induce, in contrast to the direct and absolute demands of the social choice approach. Of course, the game form approach is more general, especially when multi-valued game forms are allowed. The game form approach also permits a greater variety of ways in which rights can be respected (or violated, of course) — and even an increase in the set of (contingent) rights which it is feasible to respect.

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Other Endnotes

1. I am grateful to Jerry Kelly for a discussion which showed the need to emphasize this point.

2. Deb also defines "waiver" functions, and gives a much more extensive discussion of how both effectivity and waiver functions can be used to represent important aspects of rights.

3. Early on, Hillinger and Lapham (1971) were among those to suggest, at least implicitly, that such a framework might be necessary. They did not put forward anything like a formal framework, however.

4. Deb (1990, 1994), in his discussion of closely related issues, assumes that all such repeated hyper-rows are simply deleted from the game form.