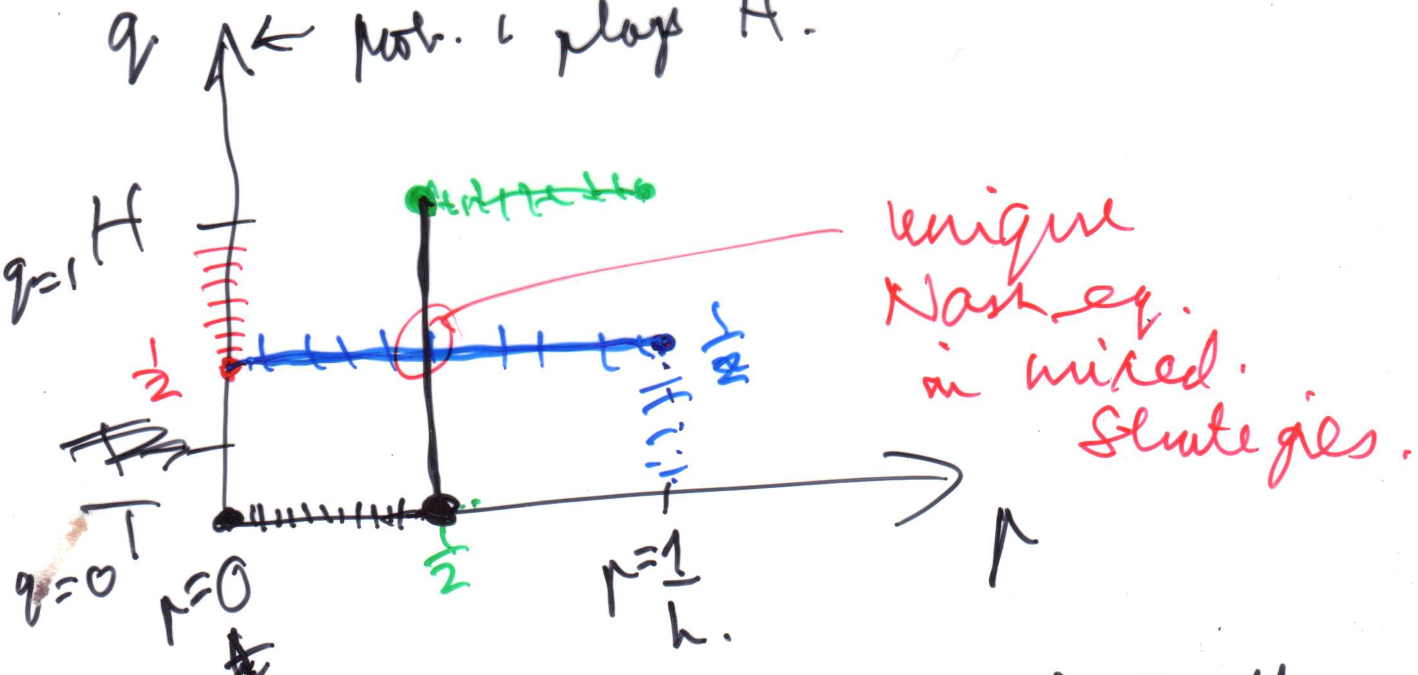
CORRESPONDENCES.

	h	t
H	1, −1	−1, 1
T	−1, 1	1, −1

Payoffs.

1 believes 2 plays h with prob. p.



T is the better response when p is small.
H " " " " " large

Best response CORRESPONDENCES.

Convex values (may) permit fixed points.

2

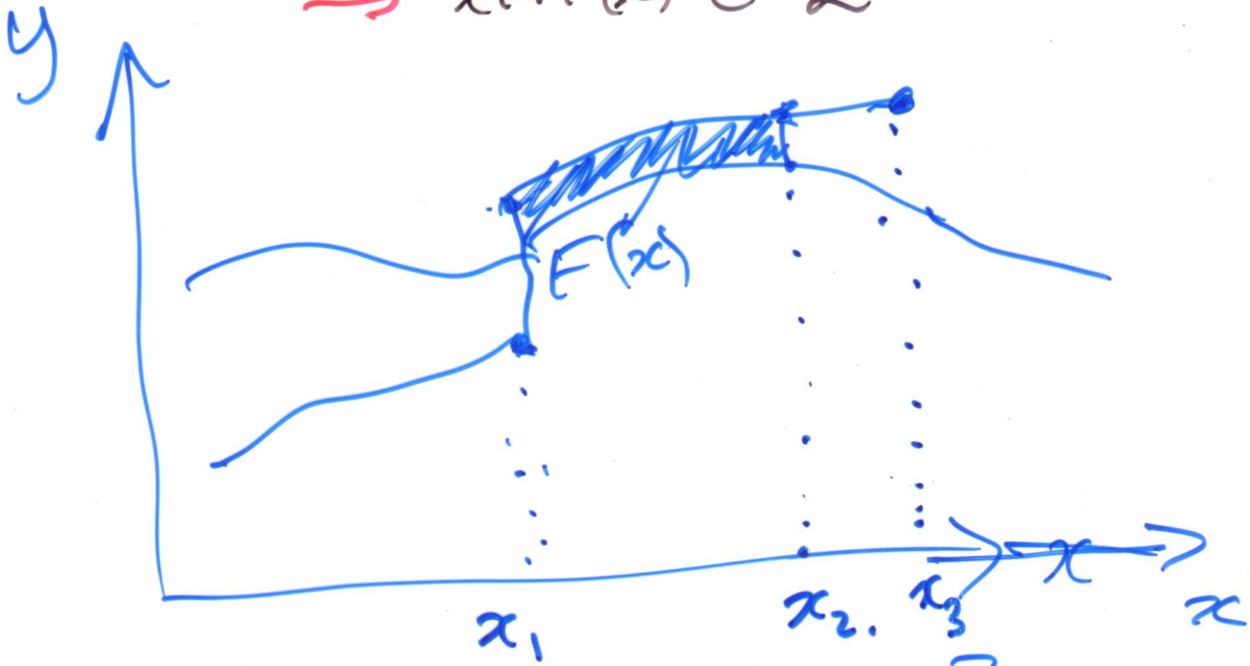
Function: $x \mapsto f(x) = y \in Y$

Single valued correspondence. $x \mapsto \{f(x)\}$

Correspondence

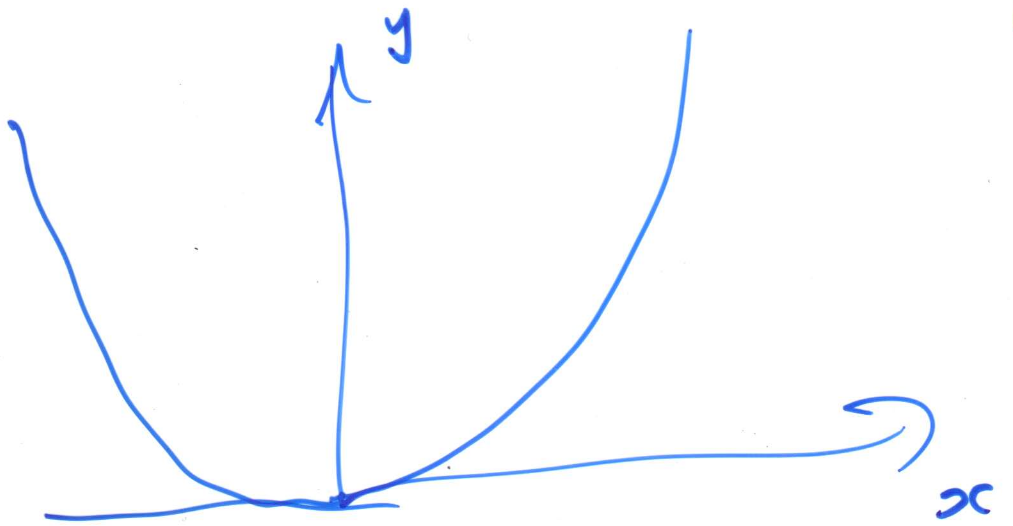
$$x \mapsto F(x) \subseteq Y$$

$$\Rightarrow x \mapsto F(x) \in 2^{Y}$$

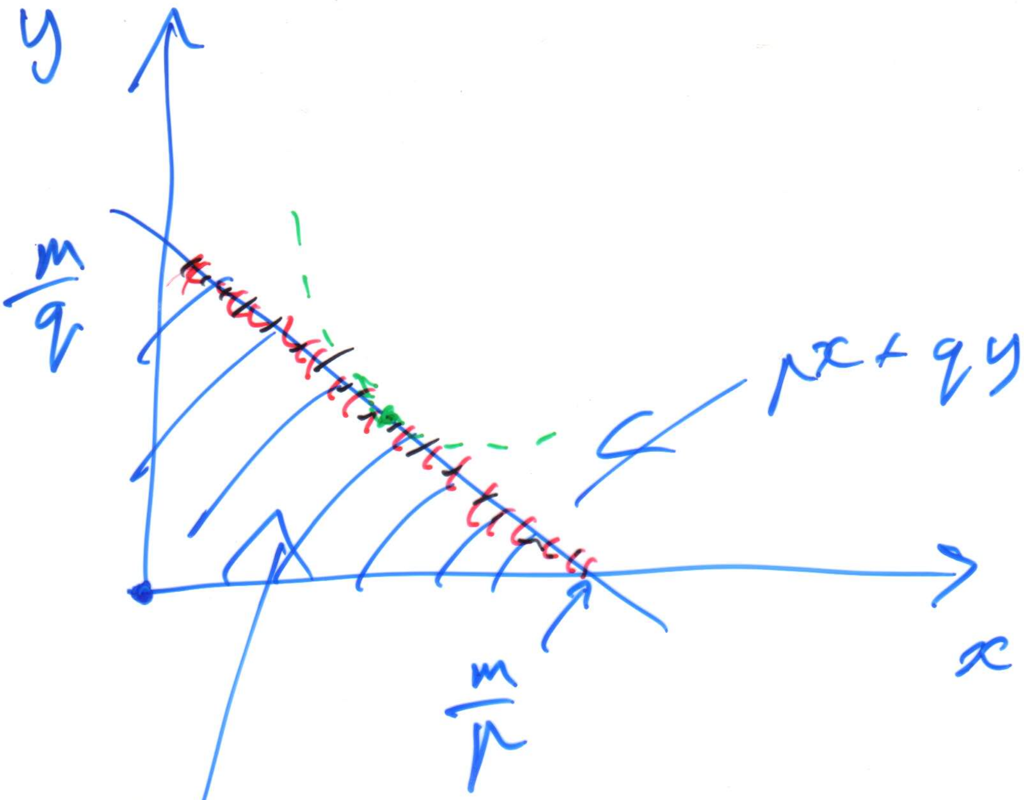


$$G = \{(x, y) \mid y \in F(x)\}$$

$y = x^2$



③



$p > 0$
$q > 0$
$m \geq 0$

$$B(p, q, m) = \{(x, y) \in \mathbb{R}^2_+ \mid px + qy \leq m\}$$

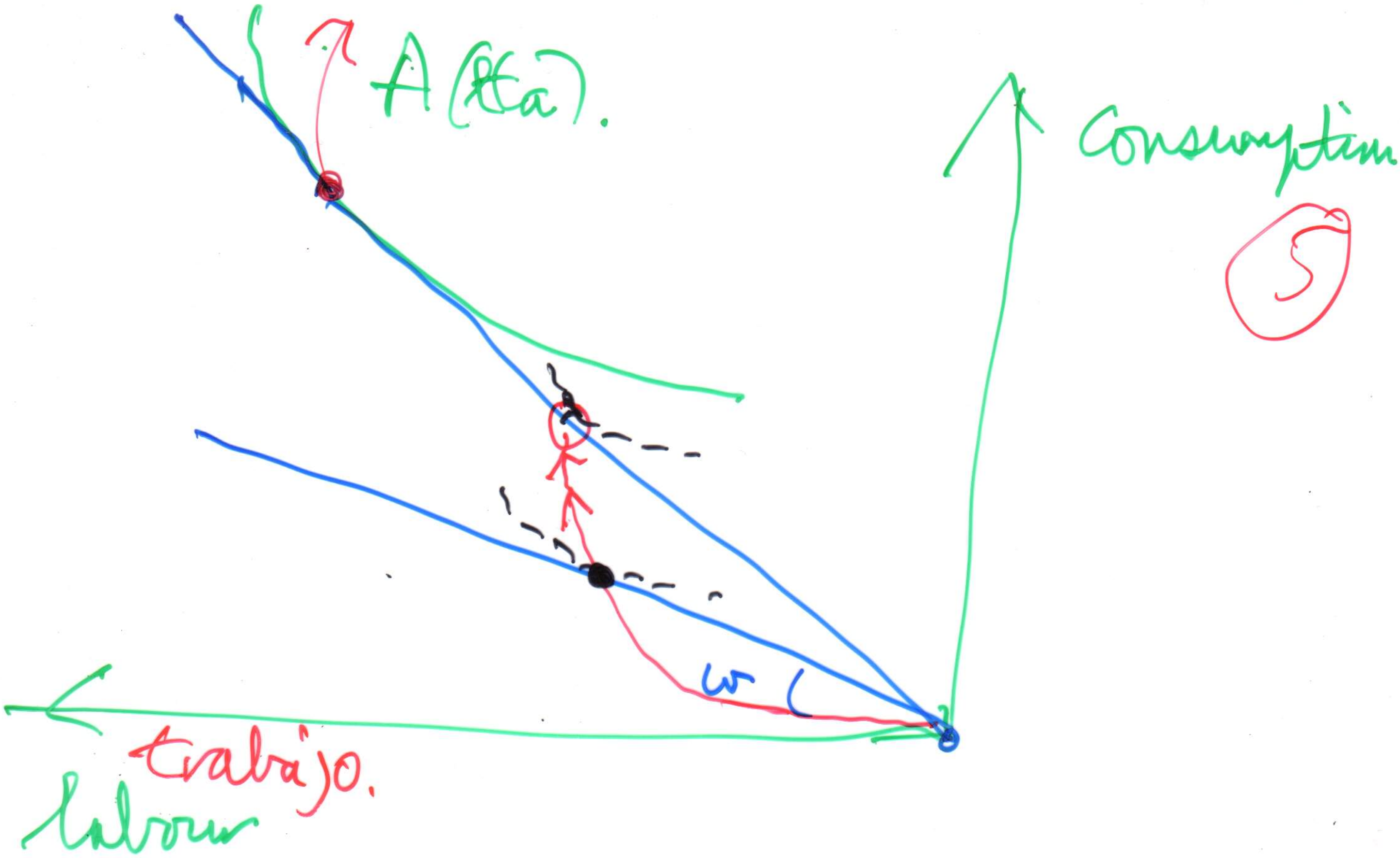
$$BL(p, q, m) = \{ \ldots \mid \ldots = m\}$$

budget line.

$$(p, q, m) \longmapsto B(p, q, m)$$

$$(p, q, m) \longmapsto (x, y)(p, q, m).$$

Single valued demands if ~~preferences are~~ utility is strictly quasi-concave



B (aja)

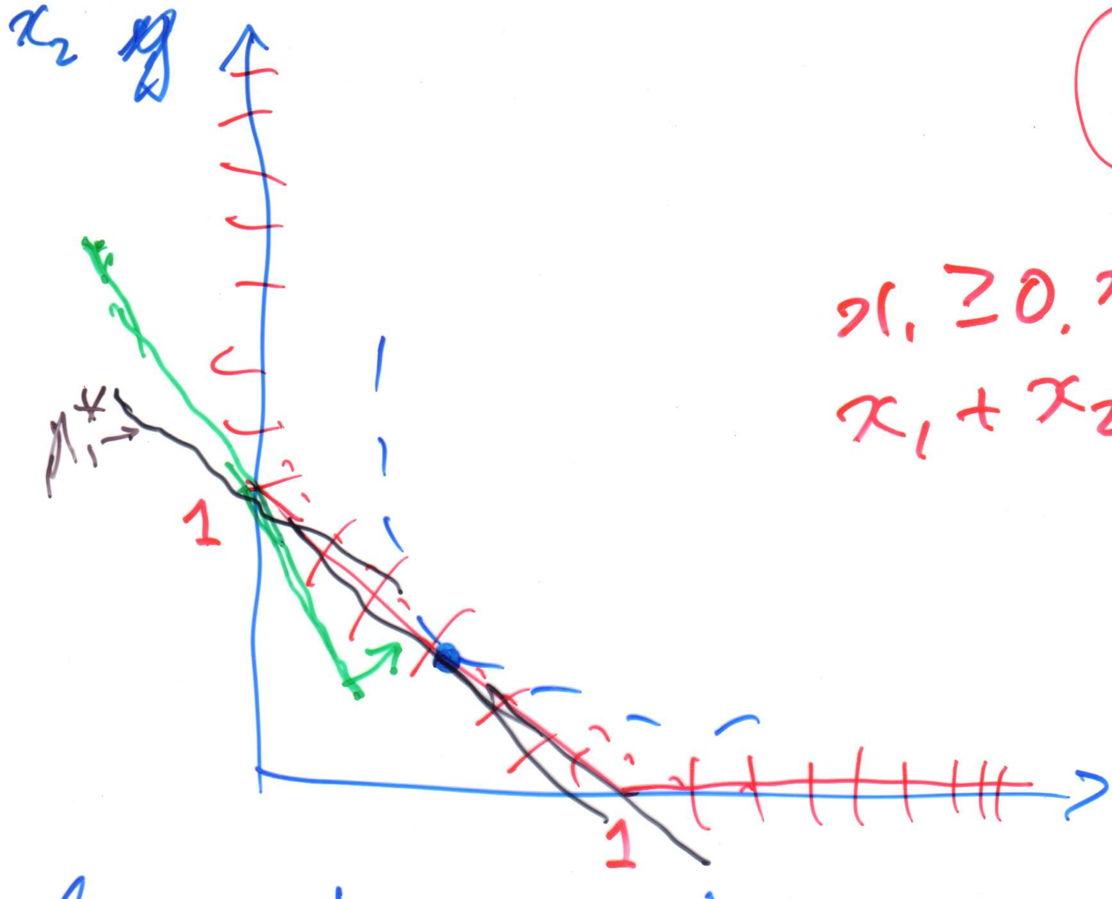
wageoffer curve jumps.

Dasgupta & Ray EJ 1985(?)

6.

$x_1 \geq 0,\ x_2 \geq 0$

$x_1 + x_2 \geq 1.$

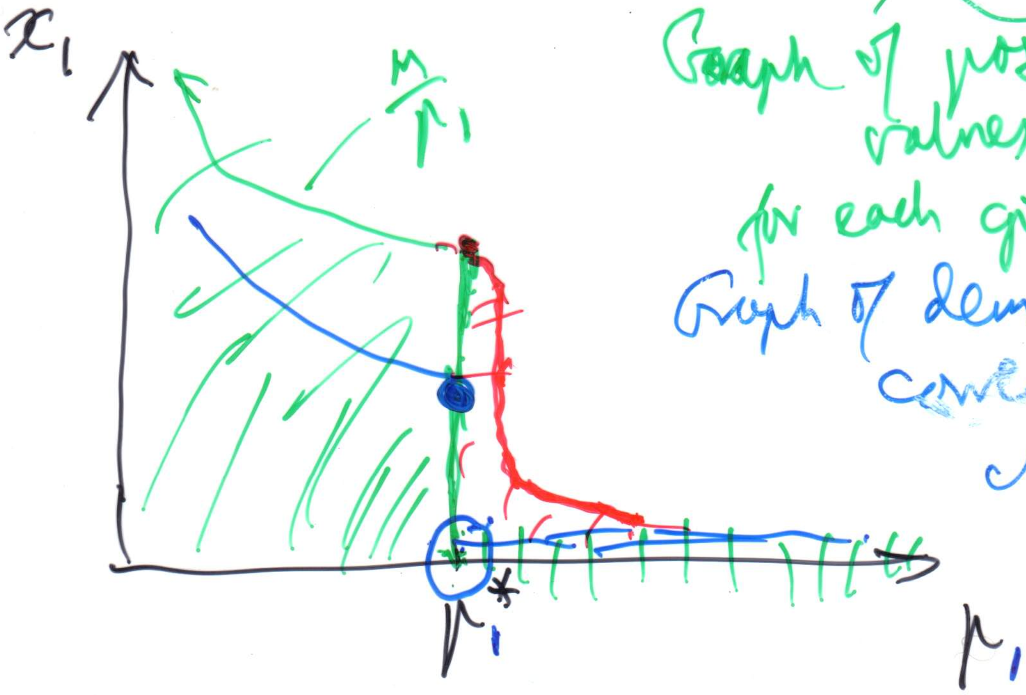


lower hemi-continuity.

uhc
not lhc.

Graph of poss. values of x_1 for each given p_1.

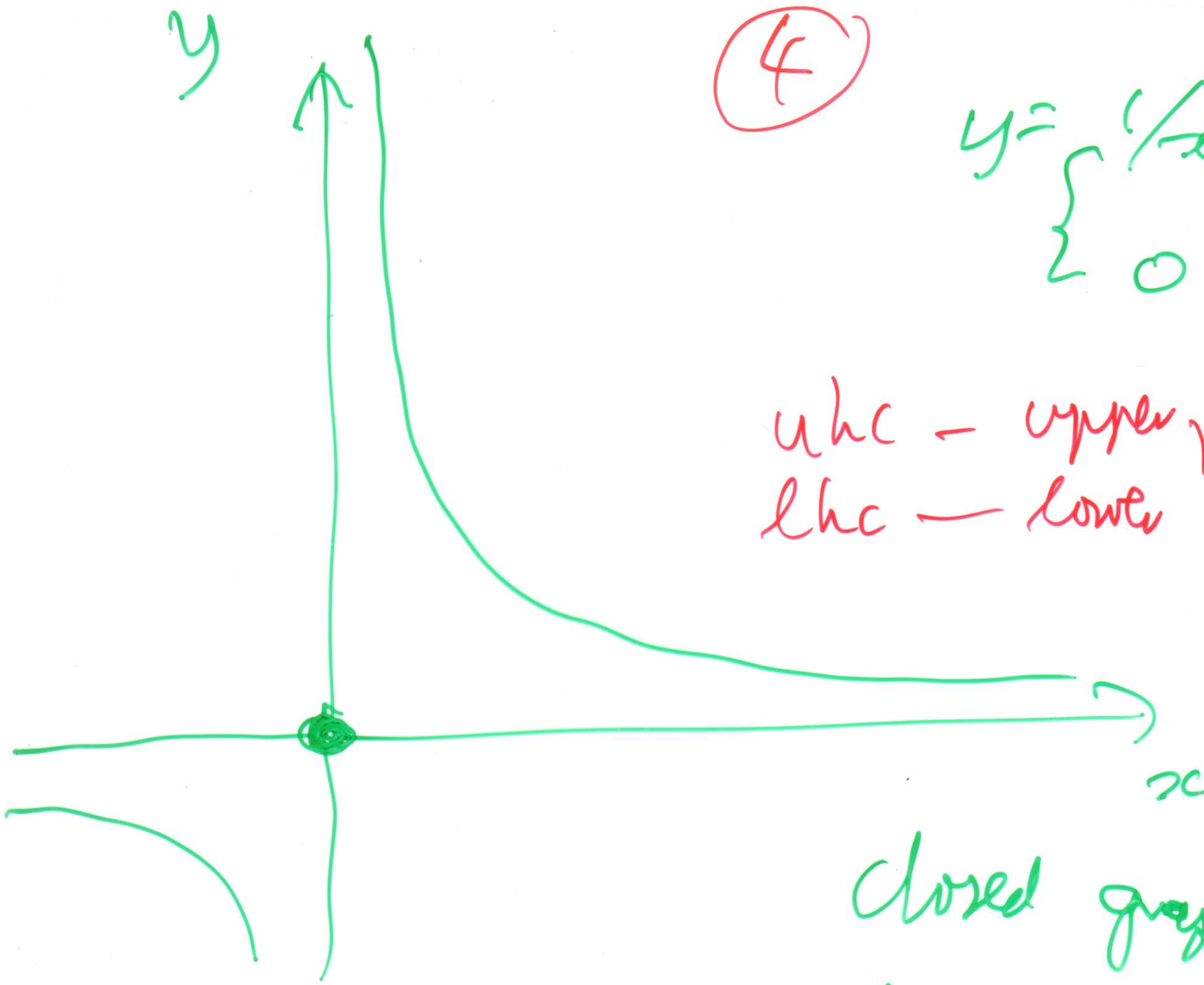
Graph of demand curve not closed.



④

$y = \begin{cases} 1/x & (x \neq 0) \\ 0 & (x = 0) \end{cases}$

uhc — upper
lhc — lower
hemi continuity



closed graph.

$x \mapsto f(x)$ is discontinuous at $x = 0$.

$x \mapsto \{f(x)\}$ fails uhc at $x = 0$.

If $(x_n, f(x_n))$ converges to (x^*, y), (with $x_n \neq x$ for enough x)
then $x \neq 0$, and $f(x_n) \to y$.
(because $y_n = \frac{1}{x_n} \to \infty$ as $x_n \to 0$)

If $(x_n, y_n) \in G$, and $(x_n, y_n) \to (x, y)$,
then $x \neq 0$, $y = 1/x$; $(x, y) \in G$.
Hence G is closed.

$x \longmapsto F(x)$ is $\boxed{lhc}$ at x^0

lower hemi-continuous

if, ~~given any~~ for all sequences $x^n \to x^0$,
and ~~any~~ all $y^0 \in F(x^0)$,
there exists $y^n \in F(x^n)$ s.t. $y^n \to y^0$

~~$x \longmapsto F(x)$ is $\boxed{uhc}$ at x^0~~
~~upper~~

~~if, for all sequences (x^n, y^n)~~
~~with $y^n \in F(x^n)$ and $x^n \to x^0$,~~
~~there is a convergent subsequence~~
~~y^{n_k} such that $y^{n_k} \to y^0 \in F(x^0)$.~~

~~AND ~~if $y^{n_k} \to$~~ if any subsequence~~
~~does converge, its limit $y^0 \in F(x^0)$.~~

closed graph property. at x^0 ~~(x^0, y^0)~~

$x \mapsto\!\!\!\to F(x)$
is uhc at x^0
if, ~~whenever~~ for all sequences $x^n \to x^0$,
and all $y^n \in F(x^n)$ $(n = 1, 2, \dots)$.
(i) if $y^n \to y^0$ then $y^0 \in F(x^0)$
(closed graph property)
(ii) y^n has a convergent subsequence.

$x \mapsto\!\!\!\to F(x)$ is uhc (resp. lhc)
if it is uhc at each $x^0 \in$ domain
(resp. lhc)

BERGE's MAXIMUM THEOREM.

Consider $\max\limits_{y} f(x, y)$ ST $y \in F(x)$
parameter

$$Y^*(x) = \arg\max_{y} \{ f(x,y) \mid y \in F(x) \}.$$

When is $Y^*(x) \neq \emptyset$?

Sufficient conditions (i) $f(x, \cdot)$ continuous w.r.t y.

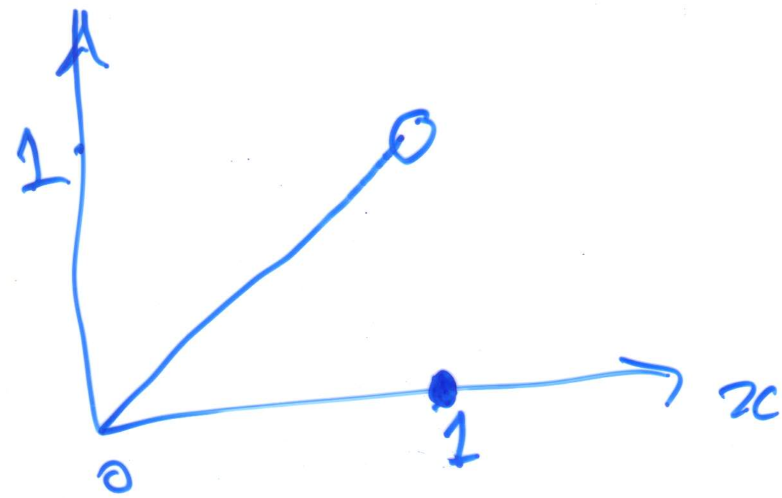
(ii) $F(x)$ compact (non-empty)

$\max -\frac{1}{x}$ ST $x \geq 1$.

closed & bounded.

$$\max_{x \in [0,1]} f(x)$$

$$f(x) = \begin{cases} x & \text{if } x < 1 \\ 0 & \text{if } x = 1. \end{cases}$$



choice corres. $Y^*(x) := \arg\max \{ f(x, y) \mid y \in F(x) \}$.

$V(x) :=$

max value function.

$f(x, y) \Longleftrightarrow y \in Y^*(x)$.

$V(x) = f(x, y)$

~~BERGE.~~ Suppose $(x, y) \mapsto f(x, y)$ is continuous ~~at x^0, for all~~ and w.r.t. y. w.r.t (x, y), and $x \mapsto\!\!\!\to F(x)$ is both uhc and lhc at x^0, with non-empty compact values. Then $x \mapsto\!\!\!\to Y^*(x)$ is uhc at x^0, and $x \mapsto V(x)$ is continuous at x^0.

BERGE'S THM.

Suppose (i) $(x,y) \mapsto f(x,y)$ is continuous

(ii) $x \mapsto\mapsto F(x)$ is both uhc & lhc at x^0 with non-empty compact values ~~throughout some neighbourhood of~~ x^0

~~(iii)~~ Then $x \mapsto\mapsto Y^*(x)$ is uhc at x^0

and $V(x) := \max \{ f(x,y) \mid y \in F(x) \}$ is continuous at x^0.

$$Y^*(x) = \arg\max \{ f(x,y) \mid y \in F(x) \}.$$

$V(x) = f(x,y)$, with $y \in F(x)$

$\Longleftrightarrow \quad y \in Y^*(x)$.

~~Define $G(x) := \{ \alpha \in \mathbb{R} \mid \alpha \leq v(x) \} = [-\infty, v(x)]$~~

~~Suppose $(x^n, \alpha^n) \in G(x)$,~~

~~Suppose $\alpha^n \in G(x^n)$ $(n=1,2,\ldots)$ and $x^n \to x^0$. Then $\alpha^n \leq f(x^n, y^n)$ $(\leq v(x^n))$ for some sequence $y^n \in F(x^n)$ $(n=1,2,\ldots)$~~